V1.2.5

International Council on Systems Engineering
A better world through a systems approach

Hamiltonian SE Foundations: Application to the Innovation Ecosystem



IW2015:
Saturday, Feb 1, 2025

INCOSE International Workshop 2025 | Seville, Spain

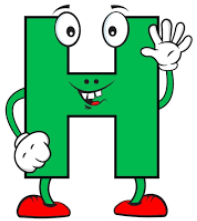
Copyright © 2025 by W. D. Schindel. Permission granted to INCOSE to publish and use

schindel@ictt.com

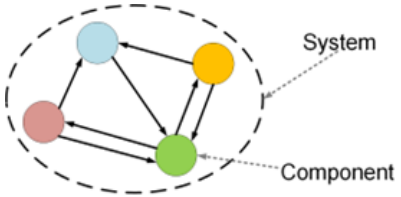


In a nutshell . . .

- W. R. Hamilton (1805-1865) provided a generalized formulation of system dynamics, based on an energy-like “characteristic function” (H) and generalized momentum (P).
- Across many applications, the Hamiltonian representation has powered a significant part of the STEM revolution to this day.
- Hamiltonians can also be applied to a much broader class of systems than systems engineers may realize—including information systems and socio-technical systems.
- Applied to the Innovation Ecosystem (the engineering and supply chain system itself), the “V&V gaps” provide potential energy terms of the Ecosystem Hamiltonian—a key to ecosystem dynamics.
- Current standards-based V&V frameworks of different disciplines are not compatible with each other—so members of four societies have been carrying out the Rosetta Stone Project, mapping them in a common representation, as a step toward shared understanding of the landscape of ecosystem potential energy.

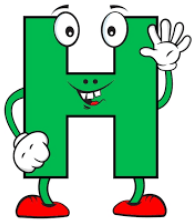


Informally, the Hamiltonian

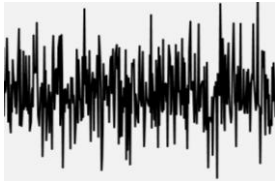


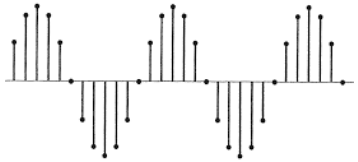
- Hamilton's energy-like "characteristic function" of a system's state is a general and mathematical way not restricted to only mechanical or other "hard" engineered systems.
- Systems: Start with a system of any type. By "system", we mean a set of interacting system components.
- Interactions: By "interact" we mean they exchange input-outputs, such as force, material, energy, or information, resulting in changes of state of the components.
- States: By "state" of a component we mean the condition of the component that can modify its current input-output behavior. Interaction thus changes state, which in turn impacts interaction.

From: Schindel, W.- (2024) Innovation ecosystem dynamics, value and learning I: What can Hamilton tell us? *Proc of the INCOSE 2024 International Symposium*, Dublin.



Non-deterministic and discrete systems

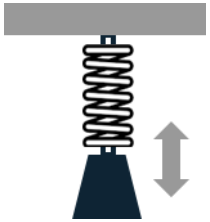




- This short and informal discussion focuses on deterministic, smoothly continuous systems, to build intuition.
- However, discrete Hamiltonian systems have been heavily explored and exploited, including providing the symplectic Hamiltonian integrators found in numerical simulation.
- Non-deterministic cases: Hamiltonian mechanics also provide the foundations of the rich historical field of statistical mechanics, where state flows are replaced by probability density flows.
- Probabilistic cases also re-enter this story through machine learning and human behavior.
- (Also separately addressed: Dissipative and nonholonomic systems)



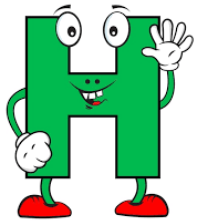
State trajectories; Hamilton's generalized momentum



- States: Have a way of representing the state of the system of interest Q(t) = {q_1,..., q_n}, whose values change over time at rate $\dot{Q}(t) = \{\dot{q}_1, \ldots, \dot{q}_n\}$, believed sufficient to characterize observed interactions.
- Characterizing System Level Behavior: Imagine now a scalar-valued function of state and time, *not yet defined here*, from Hamilton: ***H***(Q, $\dot{Q}$, t), later H(Q, P, t), intended to characterize something about the system--*we have not said how yet*.
- Momentum: Hamilton invented "generalized momentum", P(t) = {p_1,...,p_n}, generalizing the idea of momentum in elementary physics--describing ability to change $\dot{Q}$. His generalized P is defined as sensitivity of H (*not defined here just yet*) to $\dot{Q}(t)$:

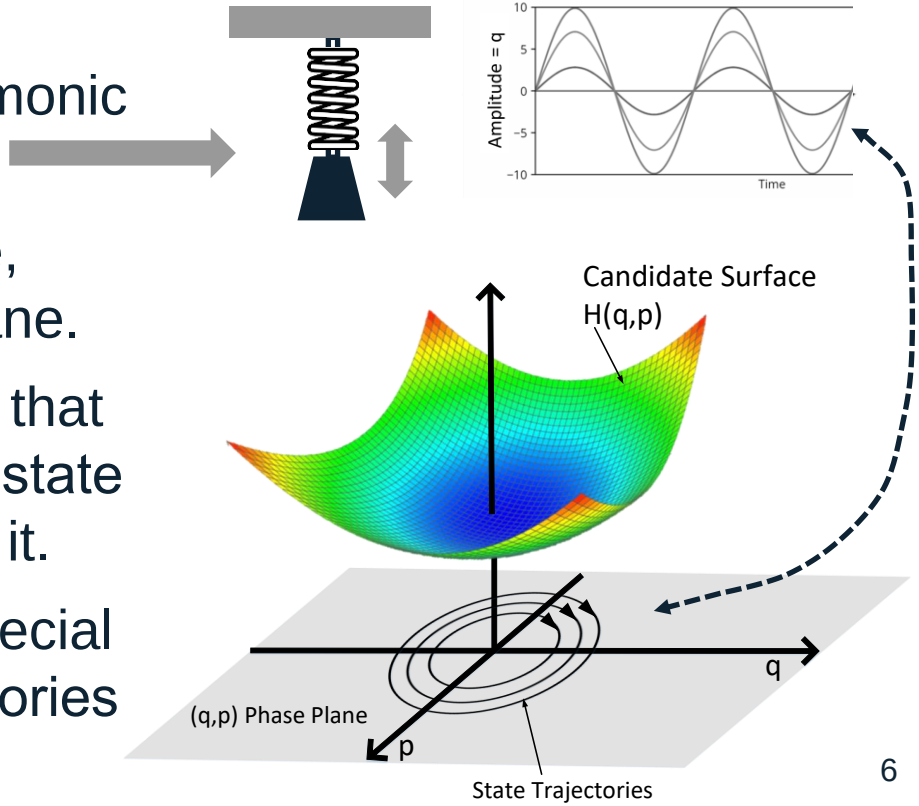
$$p_i \equiv \frac{\partial H}{\partial \dot{q}_i} \qquad \text{(Hamilton)} \qquad \text{Equation (1)}$$

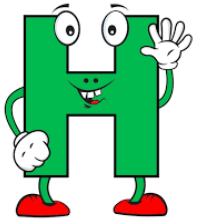
Intuition: If H turns out to be something "like" energy, this says momentum is sensitivity of energy to changes in velocity--or that energy is exchanged to change velocity.



Defining a surface, H(Q,P), that "characterizes" the family of trajectories

- Consider an undamped (simple harmonic oscillator) mass-spring system.
- We are looking for a smooth surface, H(Q, P) that sits above the (q, p) plane.
- Hamilton found a surface function H that "characterizes" the family of system state trajectories in the (q, p) plane below it.
- The Hamiltonian function has the special nature that "describes" all the trajectories below it! What could that be? . . . . .





Defining the Hamiltonian ---- H(Q,P): A different reasoning path to build intuition

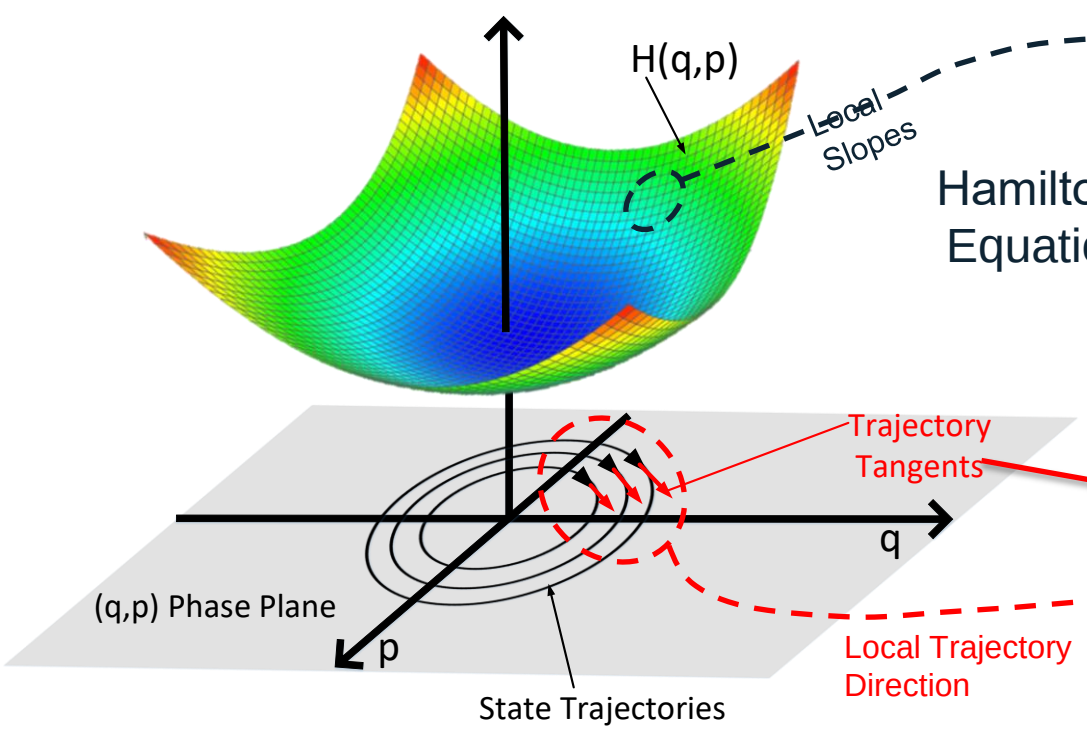
- We want H to "characterize" (describe) the system's (Q,P) trajectories, and will do so by *tying those trajectories to the local slopes of surface* H(Q,P).
- This provides an intuitive way to define H(Q,P)….
- First, pick the local sensitivity of H with respect to p_i at (q_i, p_i) is to be equal to the time rate of change of q_i along the system's state trajectory passing through (q_i, p_i):

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \text{(Hamilton)} \qquad \text{Equation (2)}$$

- Next, pick the local sensitivity of H with respect to q_i at (Q,P) is to be negative of the time rate of change of p_i along the system's state trajectory passing through (q_i, p_i):

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad \text{(Hamilton)} \qquad \text{Equation (3)}$$

- Why does it work? . . . .



Hamilton's Equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad \text{Eqn (2)}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \quad \text{Eqn (3)}$$

Trajectory Tangent Vector =

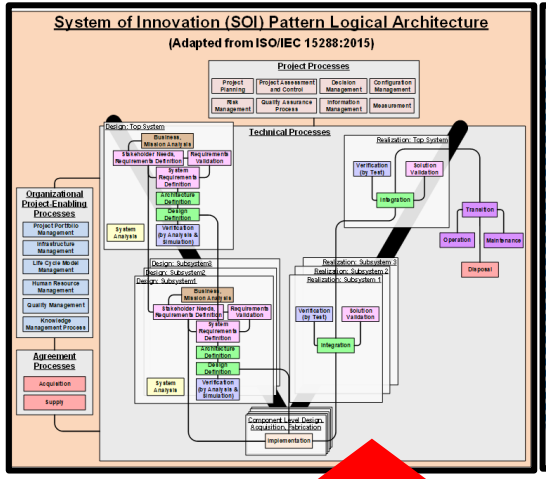
$$\{ \dot{q}_i , \dot{p}_i \} = \{ \frac{\partial H}{\partial p_i} , -\frac{\partial H}{\partial q_i} \}$$

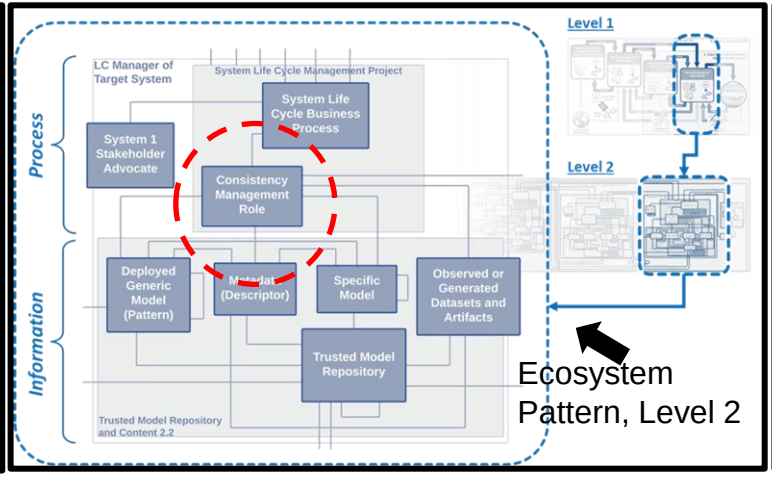
Local Trajectory Direction

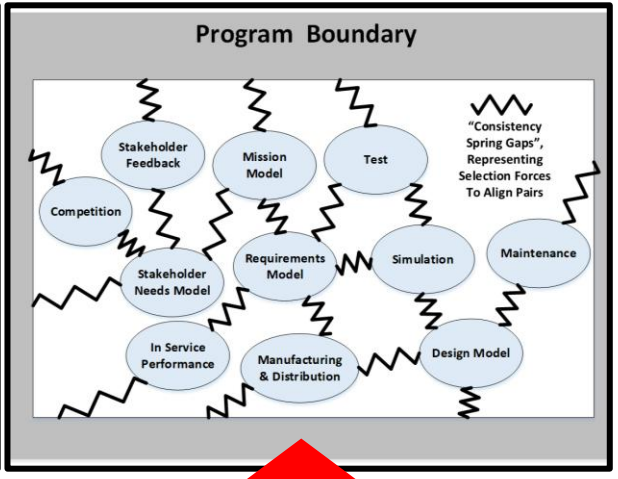
- For intuition, notice that dividing both sides of Eq (2) by both sides of Eq (3) shows that the instantaneous direction of motion in the (q, p) plane is the same as the ratio of the local surface slopes of H in the q and p directions.
- The relative slopes of H "steer" the trajectories in the (q, p) plane!
- So, the *surface* H(Q, P) "characterizes" all the dynamic *trajectories* in the (Q, P) plane.

Application: Ecosystem consistency management application

- The consistency management paradigm is about the central information thread / web running through the ASELCM pattern's representation of any engineering/life cycle management / supply chain system's primary activities.
- Including the digital thread and its many precursors: A generalization of V&V.
- Rosetta Stone Project: V&V standards map, across four technical societies.

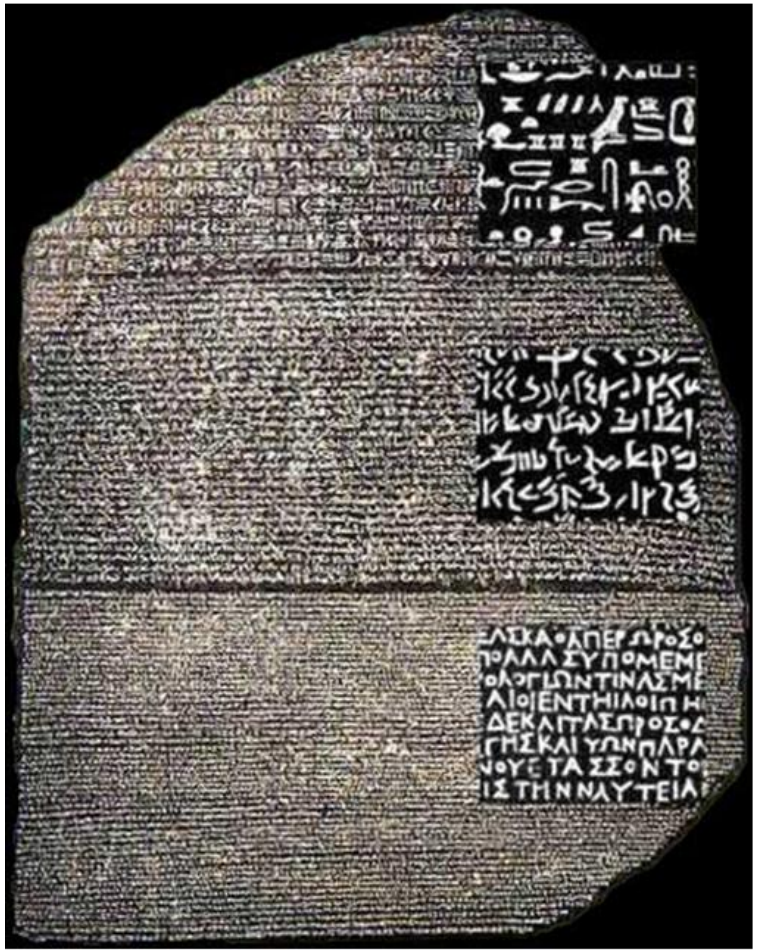






Consistency gaps = Ecosystem Hamiltonian potential energy

Rosetta Stone



What have we learned from comparing four V&V frameworks for a computational model (CM)?

- ISO 15288 (general systems, including CM)
- ASME VVUQ-1 (CM)
- NAFEMS ESQMS (CM)
- AIAA-G077 (CM)

V&V grounded in characterizing accuracy for a computational model:

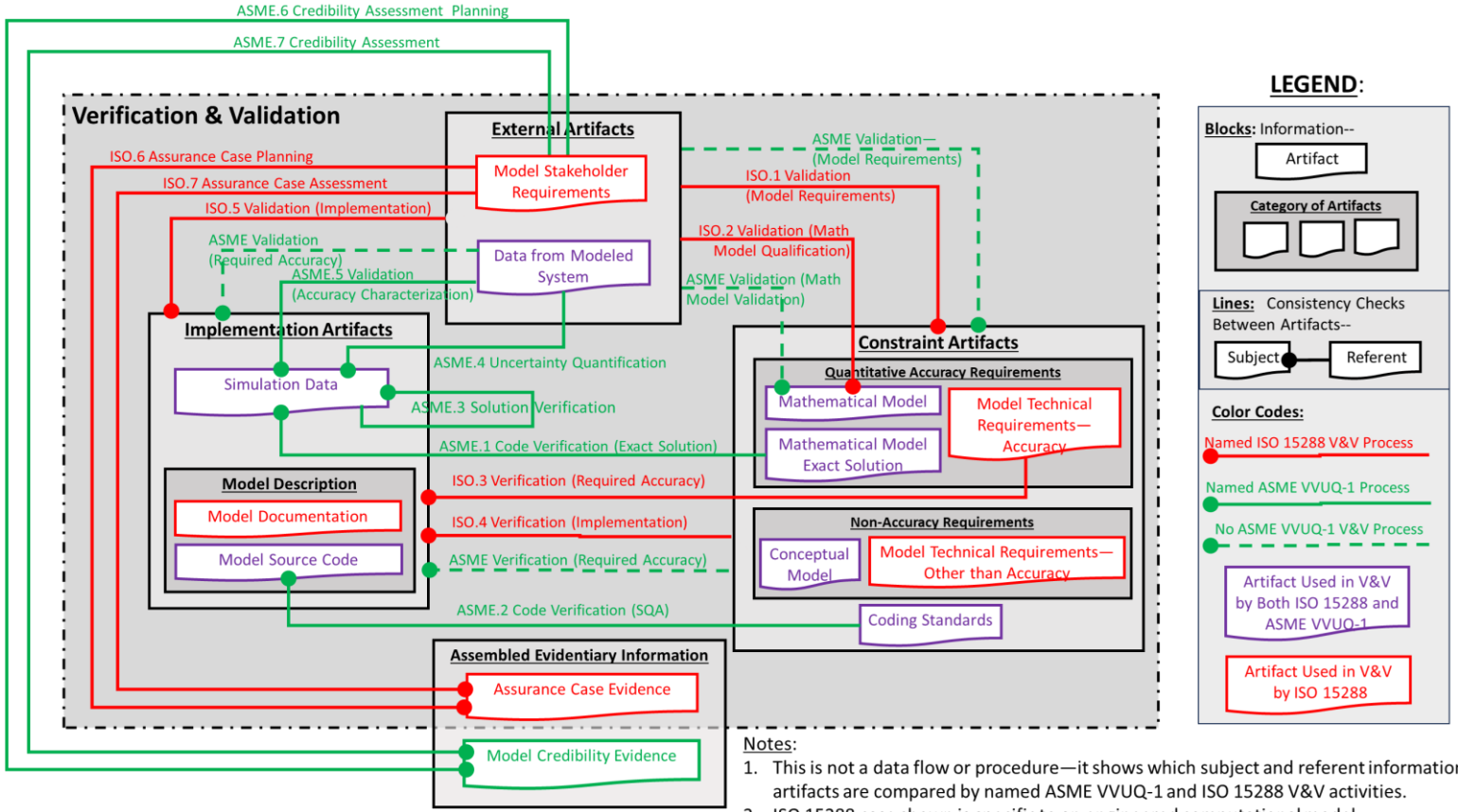
- ASME VVUQ-1
- AIAA-G077

V&V grounded in application requirements for a computational model:

- ISO 15288
- NAFEMS ESQMS

Example from Rosetta Stone Project: Comparing ISO/IEC 15288 and ASME VVUQ-1

For a copy of the 2024 project report:

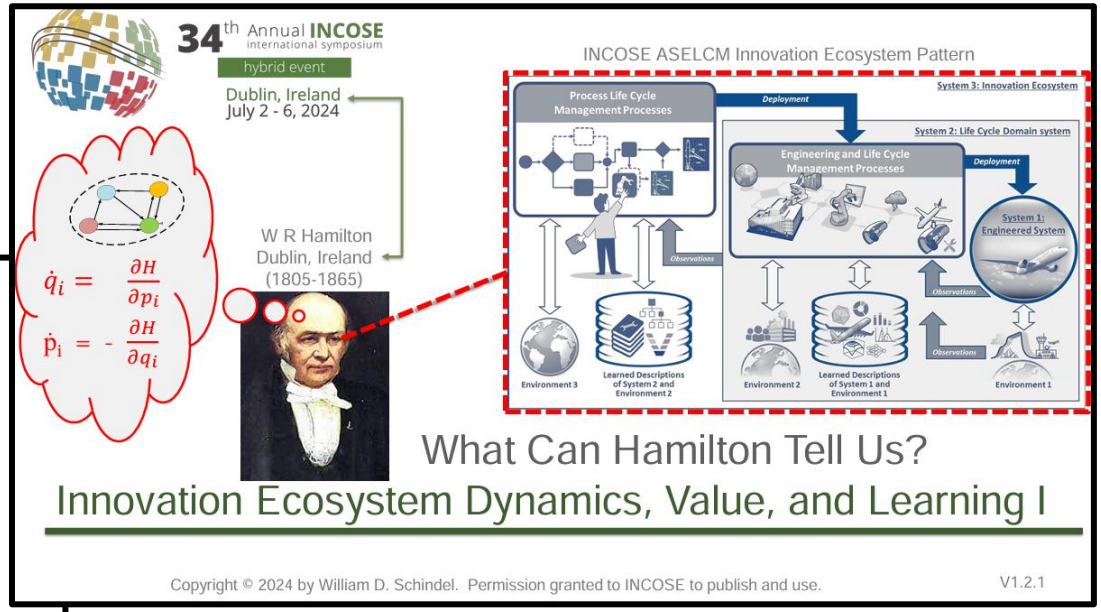


Notes:

1. This is not a data flow or procedure—it shows which subject and referent information artifacts are compared by named ASME VVUQ-1 and ISO 15288 V&V activities.
2. ISO 15288 case shown is specific to an engineered computational model.
3. Not all of ISO 15288 V&V applicabilities to that case are illustrated

References: Listed in . . .

[Download Link](#)



[Download link](#)


34th Annual INCOSE international symposium
hybrid event
Dublin, Ireland
July 2 - 6, 2024

Innovation Ecosystem Dynamics, Value and Learning I: What Can Hamilton Tell Us?

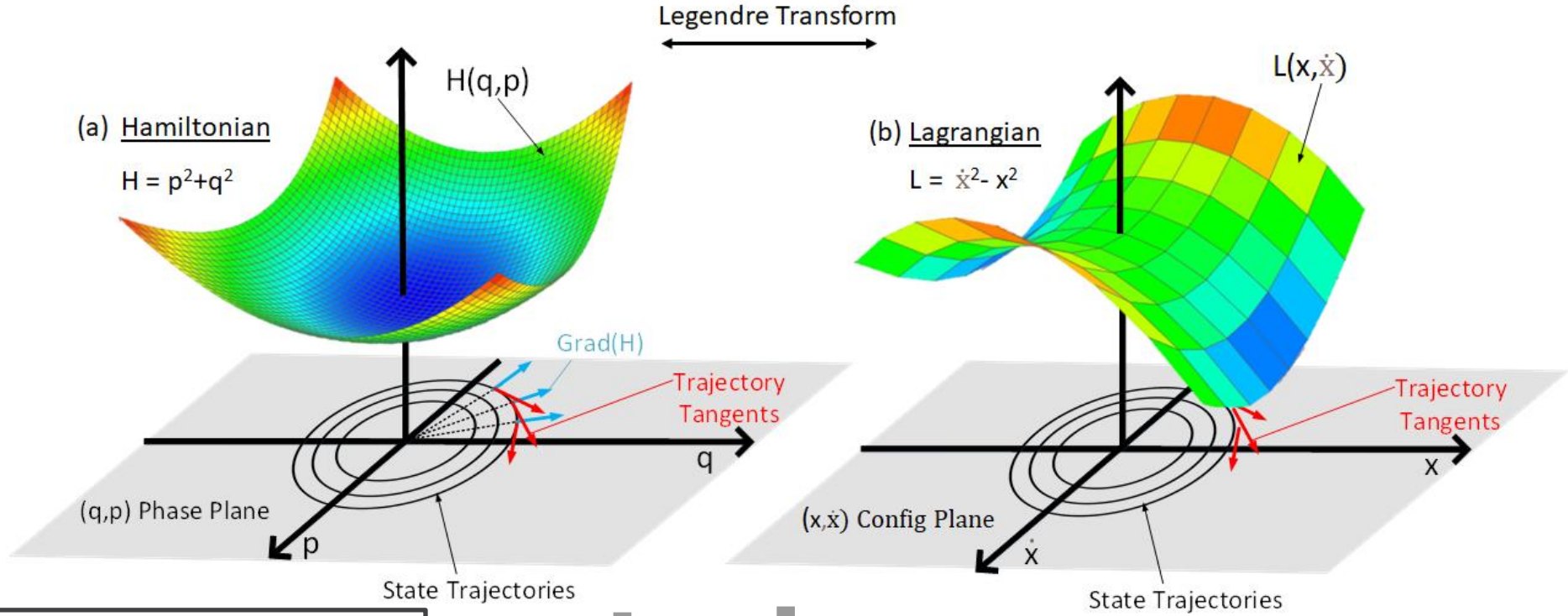
William D. Schindel
ICTT System Sciences
378 South Airport Street, Terre Haute, IN 47803 USA
812-232-2062
schindel@ictt.com

Copyright © 2024 by William D. Schindel. Permission granted to INCOSE to publish and use.

Abstract. Held in Dublin, Ireland, IS2024 invites us to refresh understanding of contributions to systems engineering by Ireland's greatest mathematician--Sir William Rowan Hamilton (1805 - 1865), Professor of Astronomy at Trinity College Dublin and Royal Astronomer of Ireland. His profound contributions to STEM deserve greater systems community attention. Supporting theory and practice, they intersect Foundations and Applications streams of INCOSE's Future of Systems Engineering (FuSE) program. Strikingly, key aspects apply to systems of all types, including socio-technical and information systems. Hamilton abstracted the energy-like generator of dynamics for all systems, while also generalizing momentum. Applied to the INCOSE Innovation Ecosystem Pattern as dynamics of learning, development, and life cycle management, this suggests an architecture for integration of the digital thread and machine learning in innovation enterprises, along with foundations of systems engineering as a dynamical system.

Keywords. Digital Thread; Hamiltonian; Hamilton's Principle; Energy; Momentum; Machine Learning;

Historically, the Lagrangian came before the Hamiltonian



$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad -\dot{p}_i = \frac{\partial H}{\partial q_i} \qquad where \;\; p_i \equiv \frac{\partial H}{\partial \dot{x}_i}$$

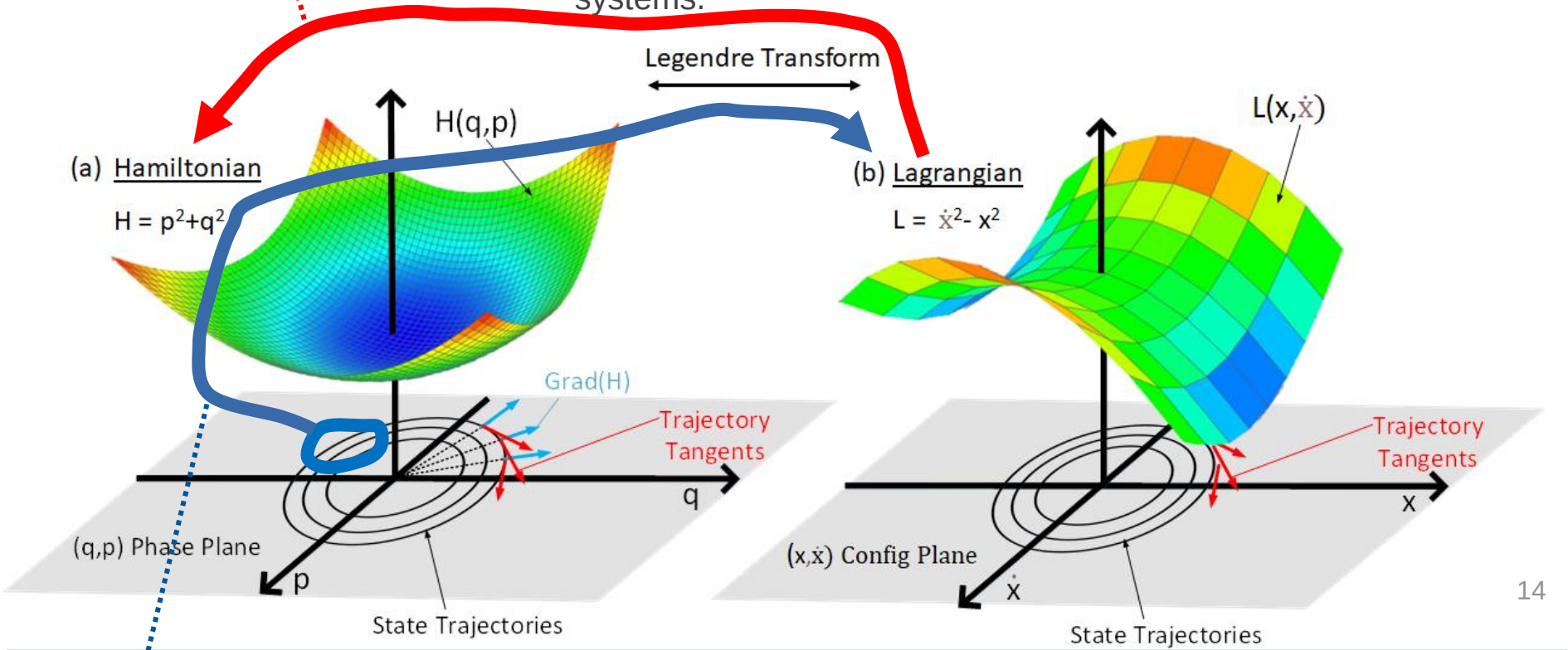
← Hamilton (first order)

Euler-Lagrange → (second order)

$$\frac{\partial L}{\partial x_i} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = 0$$

Traditional Reasoning Sequence: Assumes energy concept and leads to Hamiltonian

The historical path assumed we already had a Lagrangian and familiar mechanical or other energies. The alternate path does not require those, and teaches us about "energies" of different systems.



Alternate Reasoning Sequence: Assumes only States, Trajectories; leads directly to Hamiltonian and "energy". Examining Grad(H) leads to invariance (conservation) of H along trajectories; variational/Lagrangian insight.