V1.1.5

International Council on Systems Engineering
A better world through a systems approach

Patterns WG - System Sciences WG Collaboration



IW2015:
Saturday, Feb 1, 2025



INCOSE International Workshop 2025 | Seville, Spain

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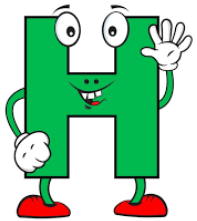
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One page summary of a SSWG – PWG collaboration

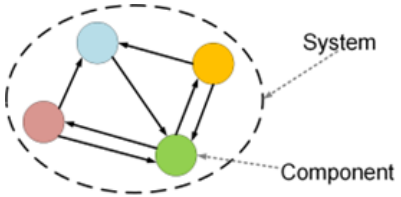
- SSWG and PWG are both interested in understanding the Innovation Ecosystem as a system in its own right—its theoretical foundations and their implications for practice.
- The SSWG and PWG are collaborating, by comparing what appear to be two different forms of representation that may be applied to the Innovation Ecosystem Pattern:
  - PWG is interested in representation based on the historical Hamiltonian of STEM.
  - SSWG is interested in representation using more recently described Category Theory.
- This collaboration also provides an opportunity to review some of the 300 years STEM revolution's history of representation of dynamical systems:
  - Are we looking for new representations before being familiar with successful historical STEM representations?
  - What insights do these representations provide? Are they re-labelings, or new understanding?

One page summary of collaboration contribution by PWG

- W. R. Hamilton (1805-1865) provided a generalized formulation of system dynamics, based on an energy-like “characteristic function” (H) and generalized momentum (P).
- In certain applications, the Hamiltonian representation has powered a significant part of the STEM revolution to this day.
- Hamiltonians can also be applied to a much broader class of systems than systems engineers may realize—including information systems and socio-technical systems.
- In application to the Innovation Ecosystem (enterprise projects), the “V&V gaps” provide potential energy terms of the Ecosystem Hamiltonian—a key to ecosystem dynamics.
- Current standards-based V&V frameworks of different disciplines are not compatible with each other—so members of four societies have carried out the Rosetta Stone Project, mapping them in a common representation, as a step toward shared understanding.

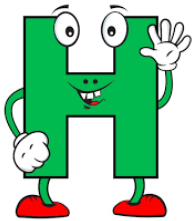


Informally, the Hamiltonian

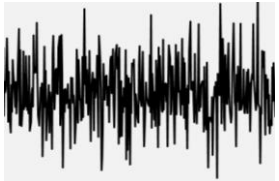


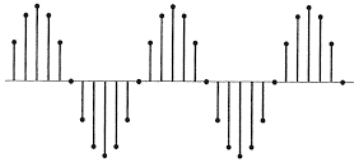
- Hamilton described an energy-like “characteristic function” of a system’s state in a general and mathematical way not restricted to only mechanical or other “hard” engineered systems.
- Systems: Start with a system of any type. By “system”, we mean a set of interacting system components.
- Interactions: By “interact” we mean they exchange input-outputs, such as force, material, energy, or information, resulting in changes of state of the components.
- States: By “state” of a component we mean the condition of the component that can modify its current input-output behavior. Interaction thus changes state, which in turn impacts interaction.

From: Schindel, W.- (2024) Innovation ecosystem dynamics, value and learning I: What can Hamilton tell us? *Proc of the INCOSE 2024 International Symposium*, Dublin.

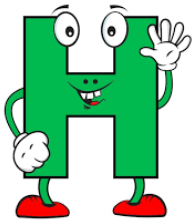


Non-deterministic and discrete systems





- This short and informal discussion focuses on deterministic, smoothly continuous systems, to build intuition.
- However, discrete Hamiltonian systems have been heavily explored and exploited, including providing the symplectic Hamiltonian integrators found in numerical simulation.
- Non-deterministic cases: Hamiltonian mechanics also provide the foundations of the rich historical field of statistical mechanics, where state flows are replaced by probability density flows.
- Probabilistic cases also re-enter this story through machine learning and human behavior.

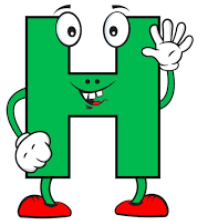


State trajectories; Hamilton's generalized momentum

- States: Have a way of representing the state of the system of interest Q(t) = {q_1,..., q_n}, whose values change over time at rate $\dot{Q}(t) = \{\dot{q}_1, \ldots, \dot{q}_n\}$, believed sufficient to characterize observed interactions.
- Characterizing System Level Behavior: Imagine now a scalar-valued function of state and time, *not yet defined here*, from Hamilton: ***H***(Q, $\dot{Q}$, t), later H(Q, P, t), intended to characterize something about the system--*we have not said how yet*.
- Momentum: Hamilton invented "generalized momentum", P(t) = {p_1,...,p_n}, generalizing the idea of momentum in elementary physics--describing ability to change $\dot{Q}$. His generalized P is defined as sensitivity of H (*not defined here just yet*) to $\dot{Q}(t)$:

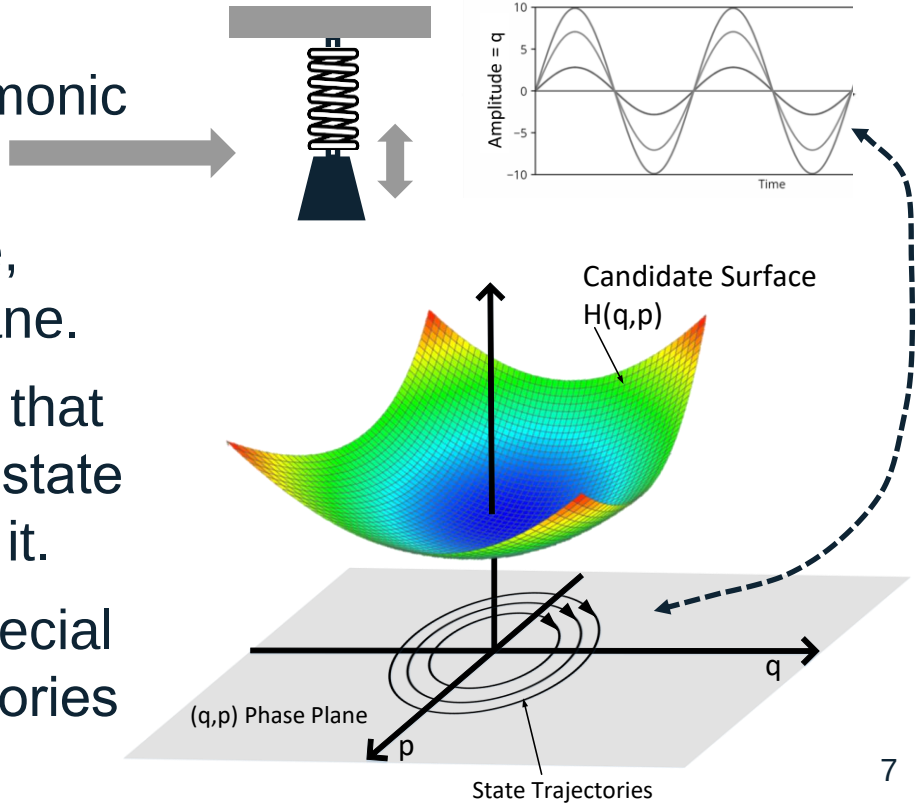
$$p_i \equiv \frac{\partial H}{\partial \dot{q}_i} \qquad \text{(Hamilton)} \qquad \text{Equation (1)}$$

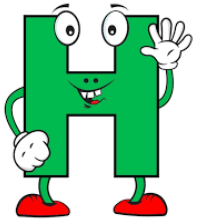
Intuition: If H turns out to be something "like" energy, this says momentum is sensitivity of energy to changes in velocity--or that energy is exchanged to change velocity.



Defining a surface, H(Q,P), that "characterizes" the family of trajectories

- Consider an undamped (simple harmonic oscillator) mass-spring system.
- We are looking for a smooth surface, H(Q, P) that sits above the (q, p) plane.
- Hamilton found a surface function H that "characterizes" the family of system state trajectories in the (q, p) plane below it.
- The Hamiltonian function has the special nature that "describes" all the trajectories below it! What could that be? . . . . .





Defining the Hamiltonian ---- H(Q,P): A different reasoning path to build intuition

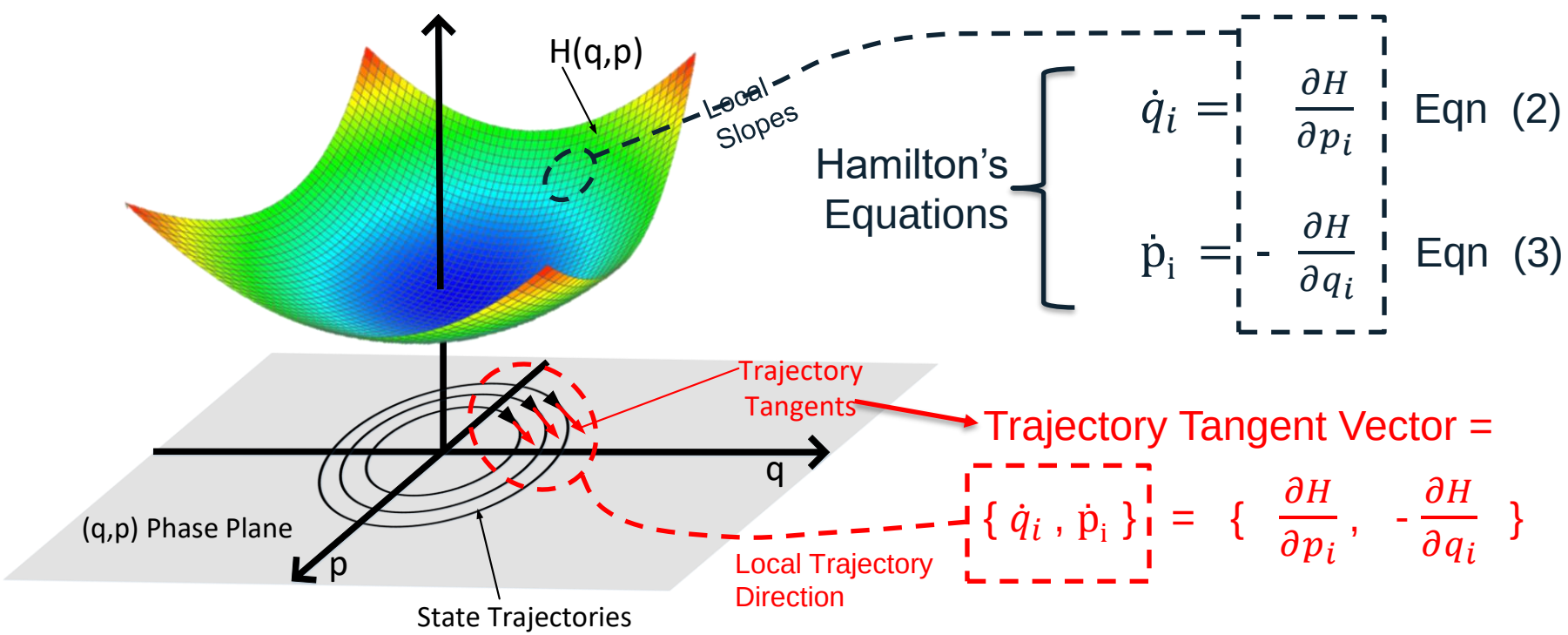
- We want H to "characterize" (describe) the system's (Q,P) trajectories, and will do so by *tying those trajectories to the local slopes of surface* H(Q,P).
- This provides an intuitive way to define H(Q,P)….
- First, pick the local sensitivity of H with respect to p_i at (q_i, p_i) is to be equal to the time rate of change of q_i along the system's state trajectory passing through (q_i, p_i):

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \text{(Hamilton)} \qquad \text{Equation (2)}$$

- Next, pick the local sensitivity of H with respect to q_i at (Q,P) is to be negative of the time rate of change of p_i along the system's state trajectory passing through (q_i, p_i):

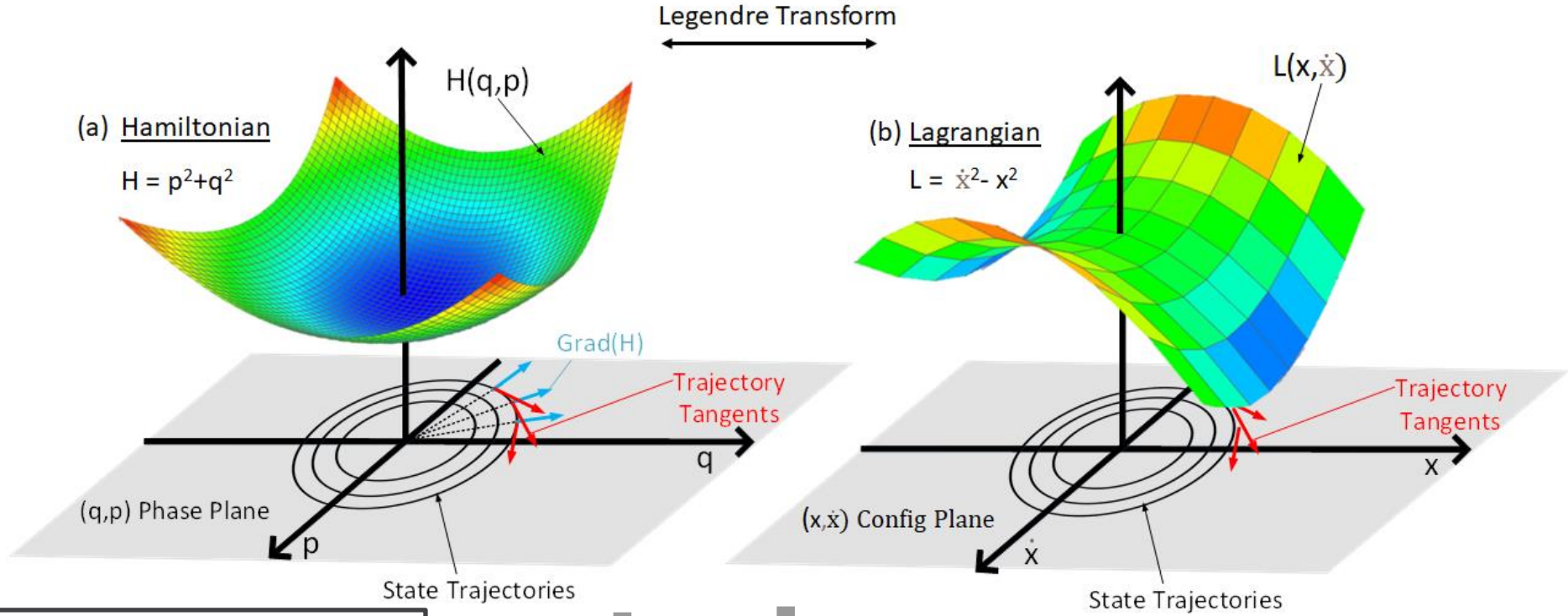
$$\dot{p}_i = - \frac{\partial H}{\partial q_i} \qquad \text{(Hamilton)} \qquad \text{Equation (3)}$$

- Why does it work? . . . .



- For intuition, notice that dividing both sides of Eq (2) by both sides of Eq (3) shows that the instantaneous direction of motion in the (q, p) plane is the same as the ratio of the local surface slopes of H in the q and p directions.
- The relative slopes of H "steer" the trajectories in the (q, p) plane!
- So, the *surface* H(Q, P) "characterizes" all the dynamic *trajectories* in the (Q, P) plane.

Historically, the Lagrangian came before the Hamiltonian



$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad -\dot{p}_i = \frac{\partial H}{\partial q_i} \qquad where \;\; p_i \equiv \frac{\partial H}{\partial \dot{x}_i}$$

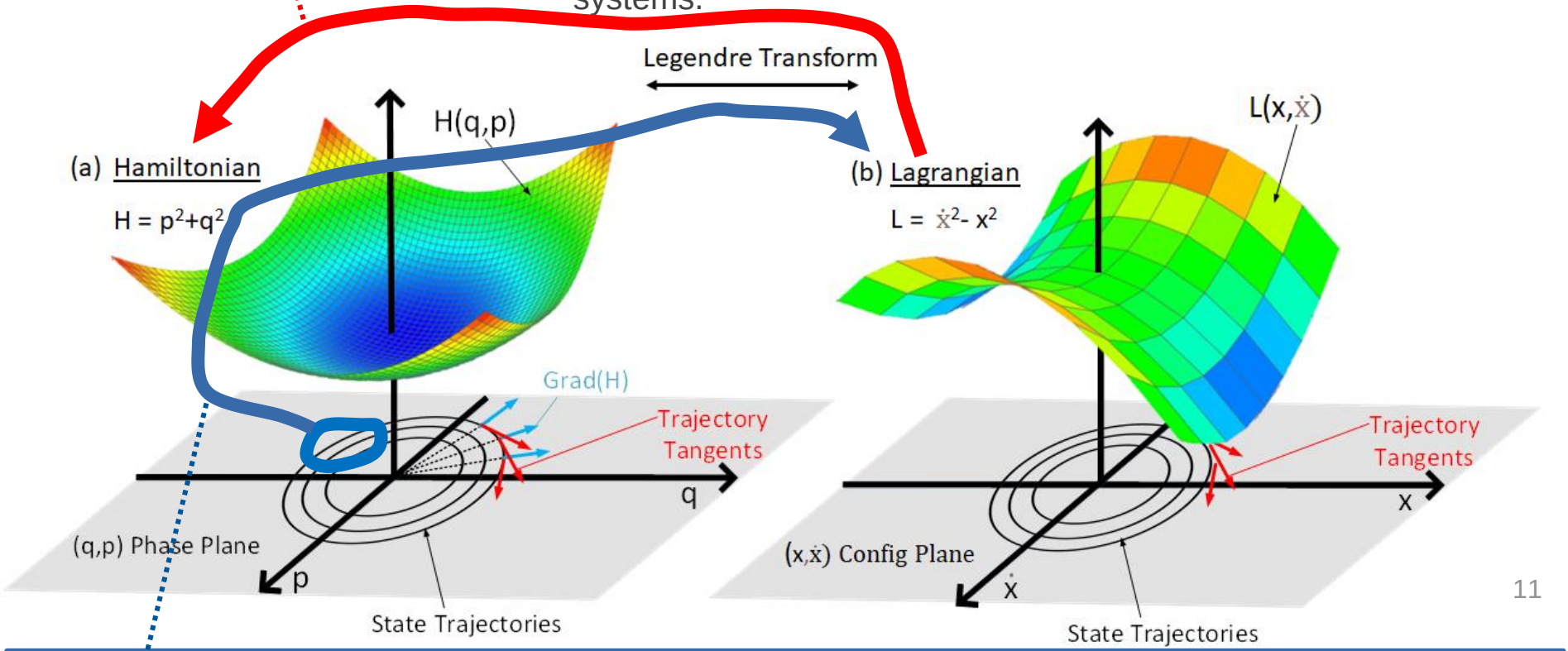
← Hamilton (first order)

Euler-Lagrange → (second order)

$$\frac{\partial L}{\partial x_i} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = 0$$

Traditional Reasoning Sequence: Assumes energy concept and leads to Hamiltonian

The historical path assumed we already had a Lagrangian and familiar mechanical or other energies. The alternate path does not require those, and teaches us about "energies" of different systems.

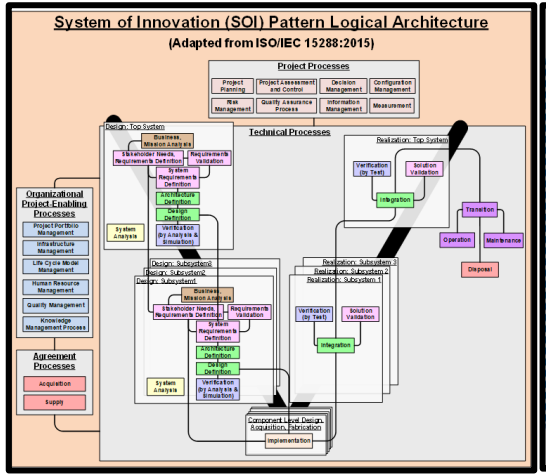


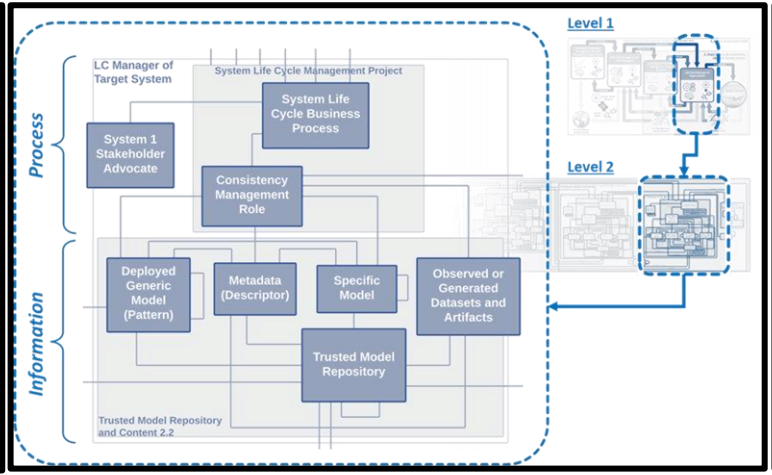
Alternate Reasoning Sequence: Assumes only States, Trajectories; leads directly to Hamiltonian and "energy". Examining Grad(H) leads to invariance (conservation) of H along trajectories; variational/Lagrangian insight.

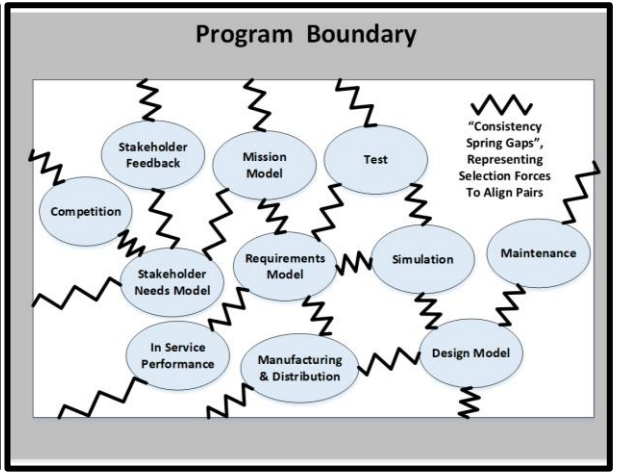
Consistency gap management paradigm

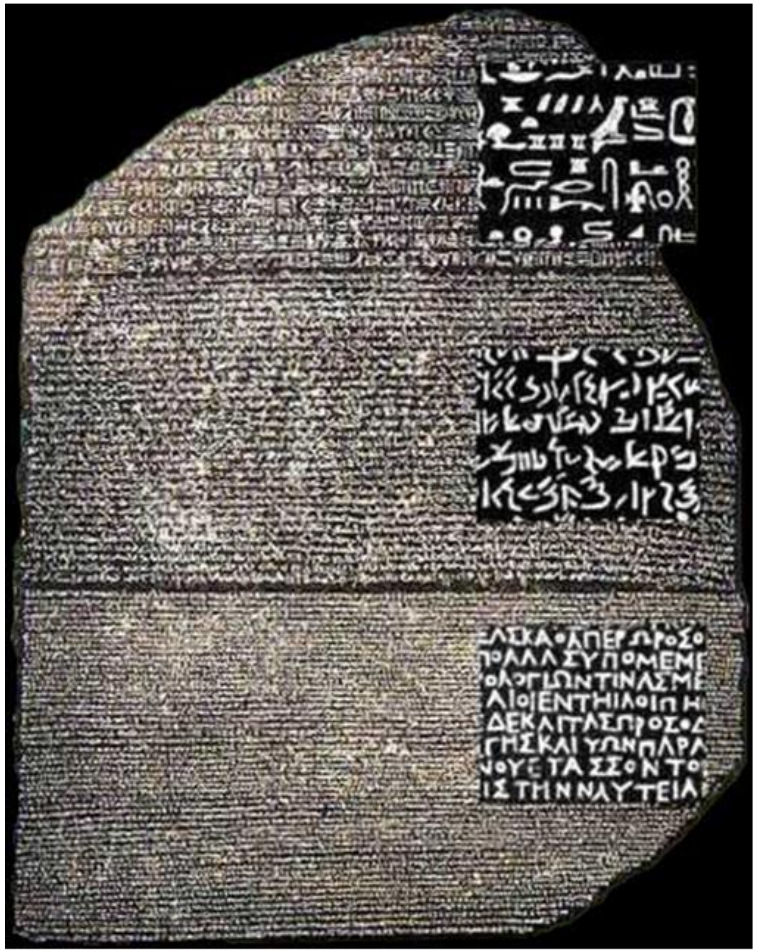
Rosetta Stone Project, across four technical societies

- The consistency management paradigm is about the central information thread / web running through the ASELCM pattern's representation of any engineering/life cycle management / supply chain system's primary activities.
- Including the digital thread and its many precursors.









What have we learned from comparing four V&V frameworks for a computational model (CM)?

- ISO 15288 (general systems, including CM)
- ASME VVUQ-1 (CM)
- NAFEMS ESQMS (CM)
- AIAA-G077 (CM)

V&V grounded in characterizing accuracy for a computational model:

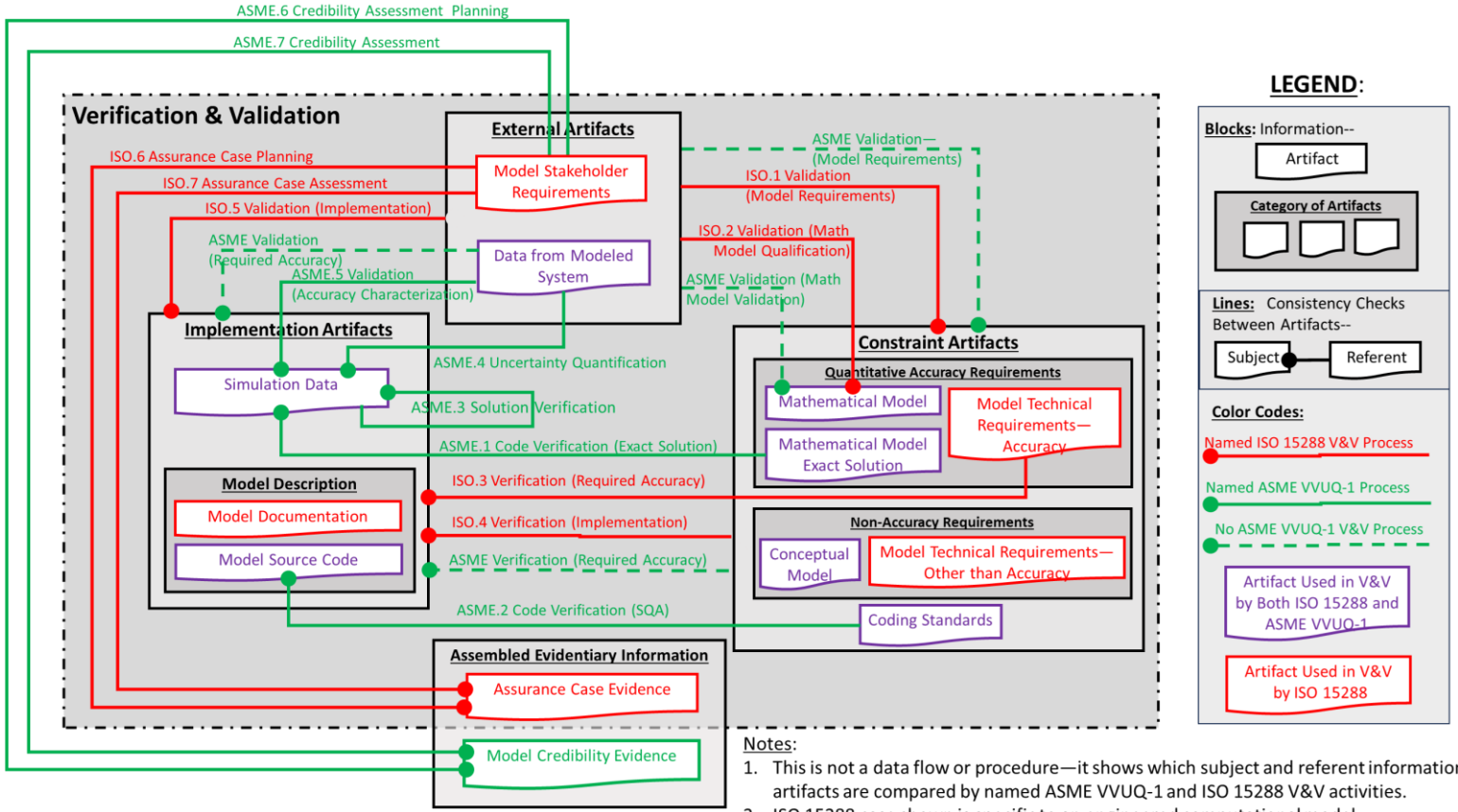
- ASME VVUQ-1
- AIAA-G077

V&V grounded in application requirements for a computational model:

- ISO 15288
- NAFEMS ESQMS

Example: Comparing ISO/IEC 15288 and ASME VVUQ-1

For a copy of the 2024 project report:



Notes:

1. This is not a data flow or procedure—it shows which subject and referent information artifacts are compared by named ASME VVUQ-1 and ISO 15288 V&V activities.
2. ISO 15288 case shown is specific to an engineered computational model.
3. Not all of ISO 15288 V&V applicabilities to that case are illustrated